Word and Excel Practice Exercise: Proposed Solution

September ##, ####

APSC-100

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Question 1: Specific Weight of Lake Ontario

Example Response

A study was conducted to determine the specific weight of lake water in Lake Ontario and to analyze how the specific weight of water affects buoyancy. The specific weight of the lake water was compared to the specific weight of ocean water ($10.1 \pm 0.1\ kN/m^3$) to examine differences in the height of objects floating in each type of water. There is a linear relationship between static pressure and depth below the surface of a fluid, for which the specific weight is the proportional constant. This relationship is given in Equation 1.

$$P = \gamma z + P_0 \quad (1)$$

where P is the pressure at a given depth (in kPa), γ is the specific weight of the fluid (in kN/m3), z is the depth below the surface of the fluid (in m), and P_0 is the pressure at the surface of the fluid (in kPa). Recently, pressure sensors were used to take pressure measurements at various depths in Lake Ontario. These measurements can be seen in Table 1.

Table 1: The recent data containing the pressure measurements at various depths below the surface.

Sensor Depth (m)	Pressure (kPa)
1	9.5
5	55
10	115.3
15	144.6
20	208.4
25	247.7
30	270.9
40	402.4
50	478.9
60	606.6

Additionally, historical data taken by an American engineer was used as another reference point to determine the specific weight. Due to the age of this data, only two data points were able to be read with certainty. These points are presented in Table 2, and were recorded by the engineer in imperial units.

Table 2: The historical pressure and depth measurements taken by the American engineer.

Sensor Depth (ft)	Sensor Depth Error (±ft)	Pressure (psi)	Pressure Error (±psi)
73	3	27.7	1.7
162	3	66.3	2.0

The data was converted to metric (SI) units such that comparison could be made to the recently collected data, as can be seen in Table 3.

Table 3: The historical pressure and depth measurements in metric units.

Sensor Depth (m)	Sensor Depth Error (± m)	Pressure (kPa)	Pressure Error (± kPa)
22.3	0.9	191	12
49.4	0.9	457	14

Both these data sets were plotted in order to determine the relationship between the pressure and the depth beneath the surface, as shown in Figure 1.

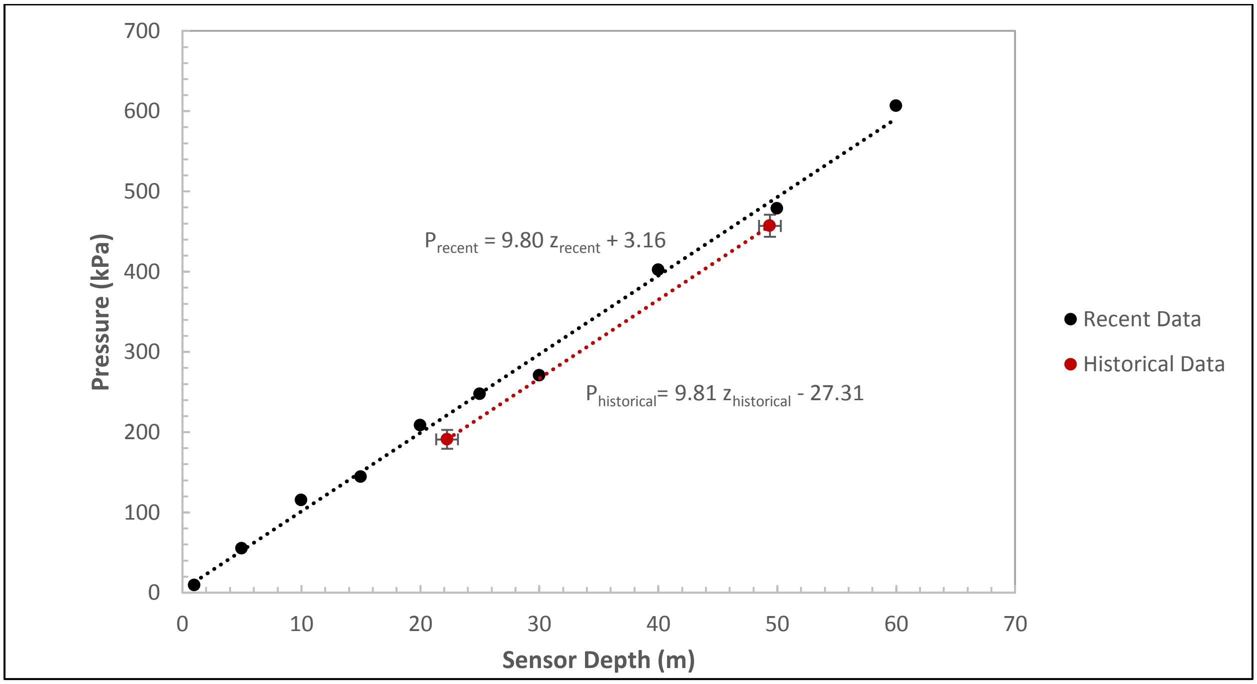

Figure 1: A plot showing the pressure distribution of Lake Ontario with respect to depth from the surface.

Linear trendlines were then added to each data series in order to determine the slopes and intercepts of the two relationships, as shown by the equations on the plot. The slopes of the relationships are equal to the specific weights of Lake Ontario for each data set. It should be noted that there appears to be a consistent difference between the historical data and the recent data in that the historical data appears to be shifted down. This discrepancy is likely due to a calibration error in the pressure measurement device used by the American engineer. The analysis of the pressure and depth measurements suggest that the specific weight of water in Lake Ontario is 9.80 kN/m^3 and 9.81 kN/m^3 for the recent and historical data sets respectively. As neither of these values agree within the limits uncertainty of the specific weight of ocean water, it can be concluded that the specific weight of Lake Ontario is less than that of ocean water. This difference can be attributed to the fact that ocean water possesses increased salt content than the fresh water in Lake Ontario, resulting in a higher specific weight.

Specific weight is the weight per unit volume whereas density is the mass per unit volume. Specific weight is therefore proportional to the density of the fluid, as given by Equation 2 [1].

$$\gamma = \rho g \qquad (2)$$

where γ is the specific weight (in N/m^3), ρ is the density (in kg/m^3), and g is the acceleration due to gravity (in m/s^2). As the buoyance force acting on a boat is proportional to both the volume of displaced water and the specific weight of the water, a boat must sit lower in Lake Ontario than the ocean due to the fact that more water must be displaced to account for the lower the specific weight of Lake Ontario.

Overview of Required Content (note to the student, not part of the sample solution)

You must include the following:

- Table of recently collected data
- Table of historical data in imperial units and converted historical data in metric (SI) units
- Scatter plot of depth vs. pressure with appropriate formatting

You should also include following information in your summary paragraphs:

- Statement of the specific weight of Ontario for both data sets ($9.80\ kN/m^3$ for the recent data and $9.81\ kN/m^3$ for the historical data)
- Subsequent conclusion that Lake Ontario specific weight is less than ocean, mainly due to the salt content of ocean water
- Explanation for lower trendline for the historical data (likely due to bad calibration of measurement device by the American engineer or other plausible explanations, use discretion)
- Explanation that density is mass per unit volume whereas specific weight is weight per unit *volume* (thus they are related by the acceleration due to gravity)
- Boat must sit lower in Lake Ontario to displace more water and make up for the lower specific weight of lake water while still creating the same force
- If an explanation of the intercept is given (not required) the student should indicate a value of zero is expected because the intercept represents atmospheric pressure which is zero when taking gauge pressure measurements.

Question 2: Resistance in a Circuit

Example Response

A simple circuit consisting of a resistive load and a power source was constructed to study the relationship between power and current and determine the resistance of the load. The current in the circuit was varied and the voltage drop across the load was measured for 0.2 second time intervals. Instantaneous power (P, in W) was determined using the relationship between voltage (V, in V) and current (I, in A) given by Equation 3.

$$P = VI = I^2R \tag{3}$$

From power (P), and knowing the time intervals (Δt, in seconds), energy dissipation (ΔE, in J) could be calculated from Equation 4.

$$\Delta E = P\Delta t \tag{4}$$

The charge flow through the resistor was also calculated for the time increments using Equation 5.

$$\Delta Q = I\Delta t \tag{5}$$

where ΔQ is the charge flow through the load (in C), I is the current passing through the load (in A), and Δt is the time increment (in seconds). A summary table was constructed to show the circuit characteristics over the course of 3.6s, as shown in Table 4.

Table 4: The circuit data over the course of 3.6s.

Point #	Elapsed Time (s)	Voltage (V)	Current (A)	ΔE (J)	ΔQ (C)	Power (W)	I^2 (A^2)
1	0.2	102.1	1.03	21.033	0.206	105.163	1.061
2	0.4	105.5	1.04	21.944	0.208	109.720	1.082
3	0.6	105.1	1.05	22.071	0.210	110.355	1.103
4	0.8	107.7	1.07	23.048	0.214	115.239	1.145
5	1	108.3	1.08	23.393	0.216	116.964	1.166
6	1.2	109.1	1.09	23.784	0.218	118.919	1.188
7	1.4	109.9	1.11	24.398	0.222	121.989	1.232
8	1.6	113.1	1.12	25.334	0.224	126.672	1.254
9	1.8	113.9	1.14	25.969	0.228	129.846	1.300
10	2	115.9	1.15	26.657	0.230	133.285	1.323
11	2.2	116	1.16	26.912	0.232	134.560	1.346
12	2.4	120.2	1.19	28.608	0.238	143.038	1.416
13	2.6	124	1.21	30.008	0.242	150.040	1.464
14	2.8	120.3	1.22	29.353	0.244	146.766	1.488
15	3	124.8	1.23	30.701	0.246	153.504	1.513
16	3.2	125.1	1.25	31.275	0.250	156.375	1.563
17	3.4	127.6	1.28	32.666	0.256	163.328	1.638
18	3.6	128.8	1.29	33.230	0.258	166.152	1.664
			Total	480.383	4.142		

The relationship given by Equation 3 was used to determine the resistance of the load through graphical and statistical methods. To determine resistance graphically, a plot of power and current measurements was constructed to determine the slope of the relationship, as shown in Figure 2.

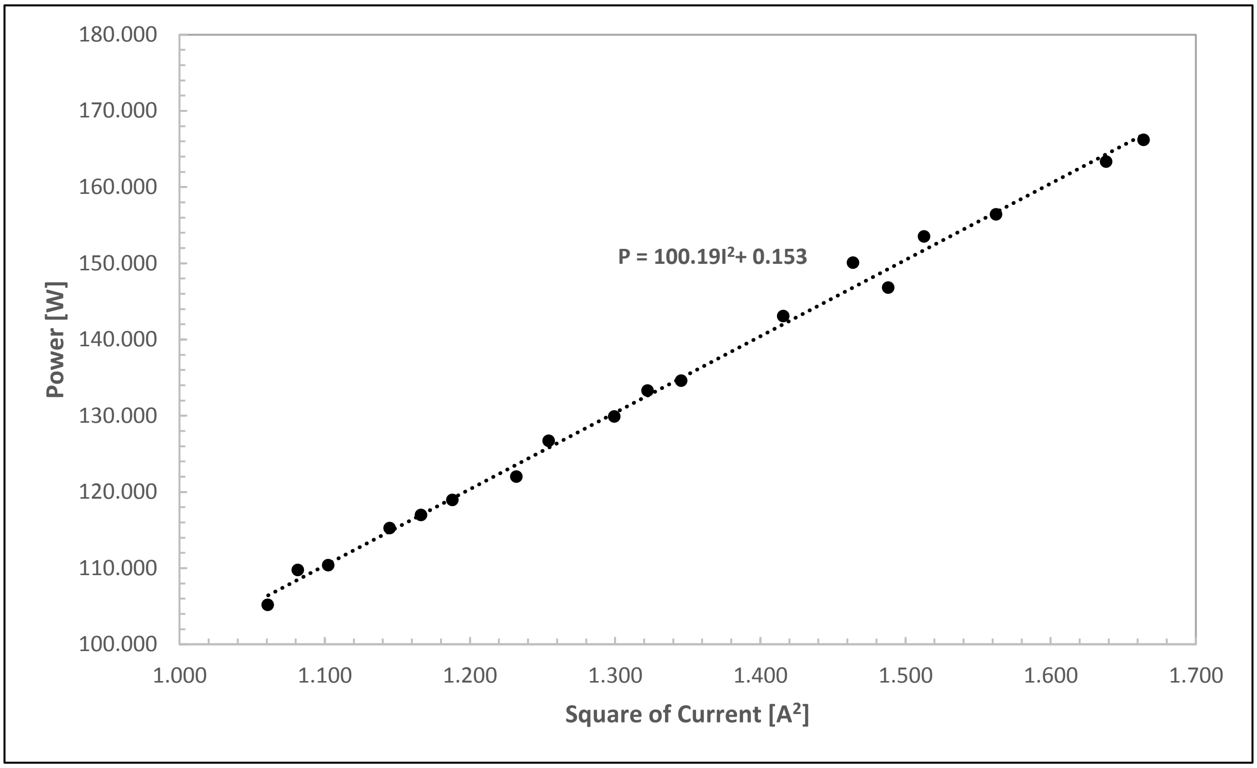

Figure 2: A plot of the instantaneous power and square of current in the circuit.

Through graphical analysis, the resistance of the load was determined from the slope to be 100.19 Ω.

A regression analysis was then performed to statistically determine the slope and intercept of the line as well as its uncertainty. The results of the regression are shown in Table 5.

Table 5: A summary of the results from a regression analysis of the circuit data..

Resistance, R (Ω)	Resistance Standard Error (±Ω)	Intercept (W)	Intercept Standard Error (±W)
100.2	1.7	0	2

In order to determine the validity of the regression, a residual plot was created to shown the normality of the residuals. It was concluded that the relationship under study was indeed linear as the residuals were normally distributed with no apparent pattern, as shown in Figure 3.

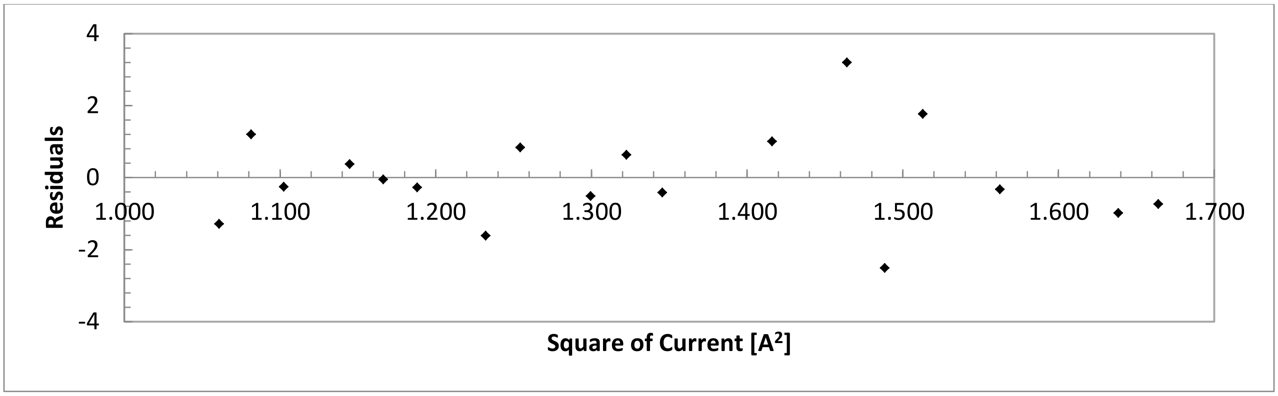

Figure 3: The residual plot for the regression analysis of the circuit.

The mean and standard error of the residuals was also examined, and the results are tabulated in Table 6. The mean of the residuals is zero within error, which is an expected result.

Table 6: The mean and standard error of the residuals is zero within error, which is the expected result.

Mean	Standard Error
0	0.3

The presence of normally distributed residuals indicates that the statistically derived resistance of $100.2 \pm 1.7\Omega$ is acceptable. The equation of the trendline shown in Figure 2, $P = 100.19I^2 - 0.153$, contains information on the intercept of the relationship. This intercept possesses no physical meaning and should be equal to zero as there can be no power dissipated in a circuit with no current passing through it. The intercept determined through regression is consistent with this theoretical value as the origin falls within its limits of uncertainty. A schematic was then constructed to depict the elements of the circuit, as shown in **Error! Reference source not found.**.

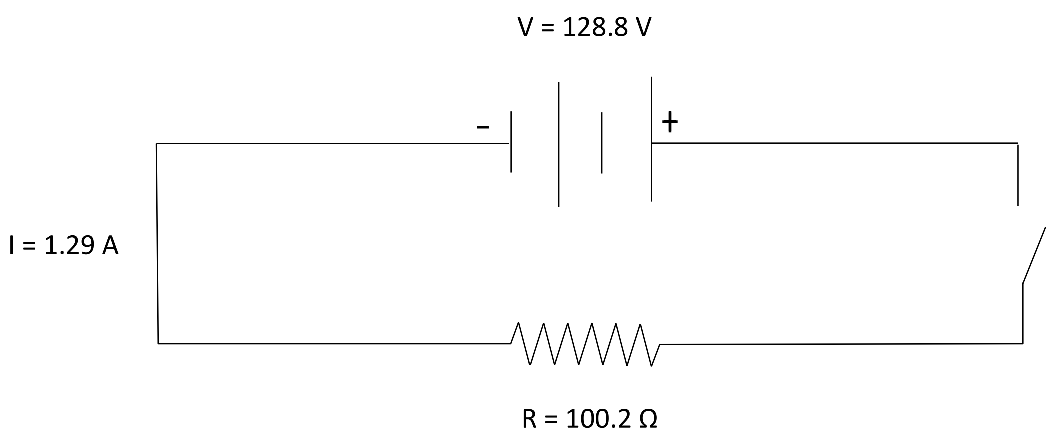

Figure 4: Schematic of circuit including voltage source and resistive load.

Overview of Required Content (note to the student, not part of the sample solution)

You must include the following:

- Completed table with $\Delta E, \Delta Q$, P and I^2
- Scatter plot of I^2 vs P with appropriate plot formatting
- Regression analysis overview including slope, y-intercept and respective standard errors
- Residual plot and table of mean and standard error of residuals
- Simple circuit schematic made in Word of the circuit based on given information

You should also include the following information in your summary paragraphs:

- Explicit statement of the fact that the intercept has no physical meaning and should be equal to zero with no measurement errors (there can be no dissipated power if there is no current flowing through the circuit)
- Explanation of how the residual plot indicates an appropriate fit
- Mean of residuals can be left in form -7.3×10^{-14} instead of 0
- The conclusion paragraph for this question can be very short as there weren't any real extensions

References

[1] Wikipedia. (2015, April 30). *Specific Weigh*t [Online]. Available: https://en.wikipedia.org/wiki/Specific_weight.